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Reg. No. : ....

Code No. : 20067 E **Sub. Code : SMMA 61**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Sixth Semester

Mathematics — Core

COMPLEX ANALYSIS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The function $f(z) = \bar{z}$ is

   (a) differentiable

   (b) nowhere differentiable

   (c) differentiable only at (0, 0)

   (d) none of these

2. If $f(z) = u + iv$ is analytic and $f(z) \neq 0$, then $\nabla^2 amp\ f(z) =$ ————

   (a) ∞ (b) 1

   (c) $\tan^{-1}\left(\frac{v}{u}\right)$ (d) 0

3. The value of $\int_c \frac{dz}{z-a}$, (c is $|z| = r$) is ————

   (a) $2\pi r$ (b) 2π

   (c) $2\pi i$ (d) 0

4. The value of $\int_c \frac{dz}{z-3}$, where c is $|z| = 2$

   (a) 0 (b) $2\pi i$

   (c) $6\pi i$ (d) 1

5. $z - \frac{z^2}{2} + \frac{z^3}{3} - \cdots (-1)^{n-1}\frac{z^n}{n} + \cdots$ represents which of following function?

   (a) $\frac{1}{1-z}$ (b) $\log(1+z)$

   (c) $\log(1-z)$ (d) $\sin z$

6. ———— is the singular point for the function $ze^{1/z}$

   (a) $z = \infty$ (b) $z = 0$

   (c) $z = 1$ (d) $z = -\infty$

7. Where we evaluated the $\int_0^{2\pi} f(\cos\theta, \sin\theta)\, d\theta$, type of integrals?

   (a) $|z| > r$ (b) $|z| < r$

   (c) $|z| = 1$ (d) $|z| = r$

8. By Jordan's Lemma, the value of $\lim_{r\to\infty} \int_c f(z)e^{iaz} dz =$ ————, where c is the semi-circle.

   (a) r (b) $-\infty$

   (c) ∞ (d) 0

9. The critical point of the transformation $w = az + b$ is ————

   (a) ± 1 (b) a

   (c) 0 (d) no critical points

10. Under the transformation $w = iz + 1$, then image of $x > 0$ is ————

    (a) $v > 0$ (b) $u > 0$

    (c) $-1 < u < 1$ (d) $v < 0$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Show that $f(z) = \sqrt{r}(\cos\theta/2 + i\sin\theta/2)$, where $r > 0$ and $0 < \theta < 2\pi$ is differentiable.

    Or

    (b) If $f(z) = u + iv$ is analytic and $f(z) \neq 0$ then prove that $\nabla^2 \log|f(z)| = 0$.

12. (a) Evaluate $\int_c \frac{z+2}{z} dz$, where c is the semi circle $z = 2e^{i\theta}$, where $0 \leq \theta \leq \pi$.

    Or

    (b) State and prove Liouville's theorem.

13. (a) Find the Taylor series to represent $\frac{z^2-1}{(z+2)(z+3)}$ in $|z|<2$.

Or

(b) Use Laurent's series to find the residue of $\frac{d^{2z}}{(z-1)^2}$ at $z=1$.

14. (a) Evaluate $\int_0^{2\pi}\frac{d\theta}{5+4\sin\theta}$.

Or

(b) Evaluate $\int_0^{\infty}\frac{dx}{x^2+1}$.

15. (a) Find the image of the circle $|z-3|=5$ under the transformation $w=\frac{1}{z}$.

Or

(b) Prove that a bilinear transformation $w=\frac{az+b}{cz+d}$, where $ad-bc\neq 0$ maps the real axis into itself iff a,b,c,d are real.

PART C — ($5\times 8=40$ marks)

Answer ALL questions choosing either (a) or (b).

16. (a) State and prove $C-R$ equations in polar form.

Or

(b) Prove that $u=2x-x^3+3xy^2$ is harmonic and find its harmonic conjugate. Also find the corresponding analytic function.

17. (a) State and prove Cauchy's theorem.

Or

(b) Evaluate $\int_c\frac{e^z}{(z+2)(z+1)^2}$ where c is $|z|=3$.

18. (a) Expand : $f(z)=\frac{z}{(z-1)(z-2)}$ in a Laurent's series valid for (i) $|z|<1$ (ii) $1<|z|<2$ (iii) $|z|>2$.

Or

(b) Use Cauchy residue theorem, to evaluate $\int_c\frac{3z^2+z-1}{(z^2-1)(z-3)}dz$, around the circle $|z|=2$.

19. (a) Prove that $\int_0^{2\pi}\frac{d\theta}{1+a\sin\theta}=\frac{2\pi}{\sqrt{1-a^2}}$, $(-1<a<1)$.

Or

(b) Evaluate : $\int_{-\infty}^{\infty}\frac{\cos x}{(x^2+a^2)(x^2+b^2)}dx$, $(a>b>0)$.

20. (a) Find the bilinear transformation which maps $-1, 0, 1$ of the z-plane onto $-1, -i, 1$ of the co-plane.

Or

(b) Prove that a bilinear transformation preserves inverse points.